CryptoVerif: Mechanising Game-Based Proofs

Part II

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Recap: CryptoVerif Formalizes Game-Based Proofs

encode security properties here

initial game $\underbrace{\rightarrow \cdots \rightarrow}_{\text{transformations}}$ intermediate games $\underbrace{\rightarrow \cdots \rightarrow}_{\text{transformations}}$ final game

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- CryptoVerif *constructs* a sequence of computationally indistinguishable games
- built-in proof strategy, and detailed guidance by user
- supports indistinguishability, secrecy, authentication properties
- computes exact security probability bound

What to Expect from Part II

A more complex example, a protocol with multiple messages:
Signed Diffie-Hellman, a 2-party Authenticated Key Exchange protocol

What's new?

- model a hash function as a random oracle
- use a Computational Diffie-Hellman (CDH) assumption
- prove key secrecy in a protocol
- prove authentication properties using correspondences between events
- model a Public-Key Infrastructure using a list (`table` in CryptoVerif)

Cryptographic Building Blocks

Cryptographic Building Block: Hash Function

Hash Function
$\mathsf{hash} : \{0,1\}^* \rightarrow \{0,1\}^{\mathsf{hashlen}}$.

Example:

$k \leftarrow \mathsf{hash}(m)$

Intuition: for different inputs, outputs are uniformly random and independent of each other.

Cryptographic Building Block: Signature

Cryptographic Signature

$sk, pk \xleftarrow{\$} keygenSig()$

$\sigma \xleftarrow{\$} \mathsf{sign}(m, sk)$

$b \leftarrow \mathsf{verify}(m, pk, \sigma)$ returns 1 iff σ is a correct signature

Intuition: it is hard to forge a signature

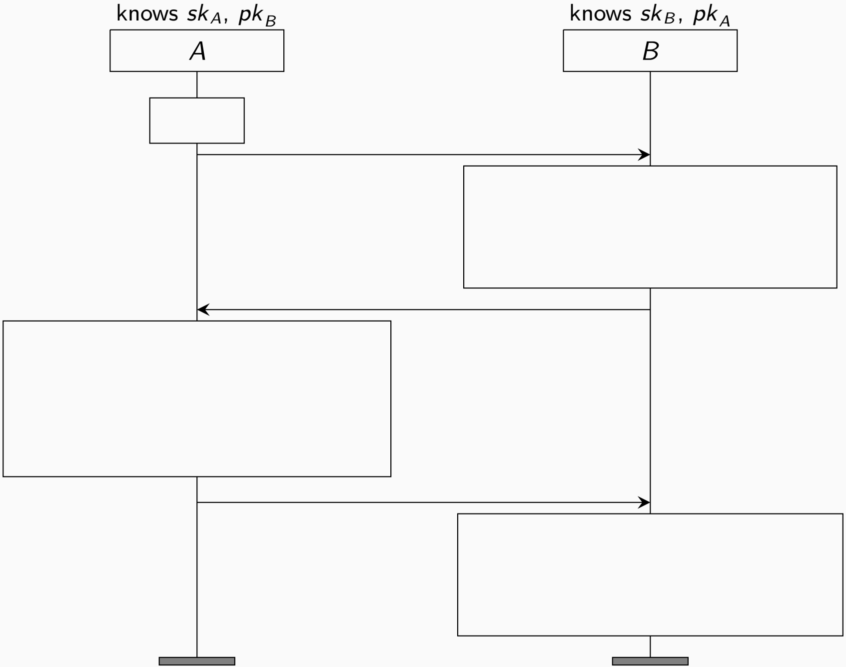
Cryptographic Building Block: Diffie-Hellman

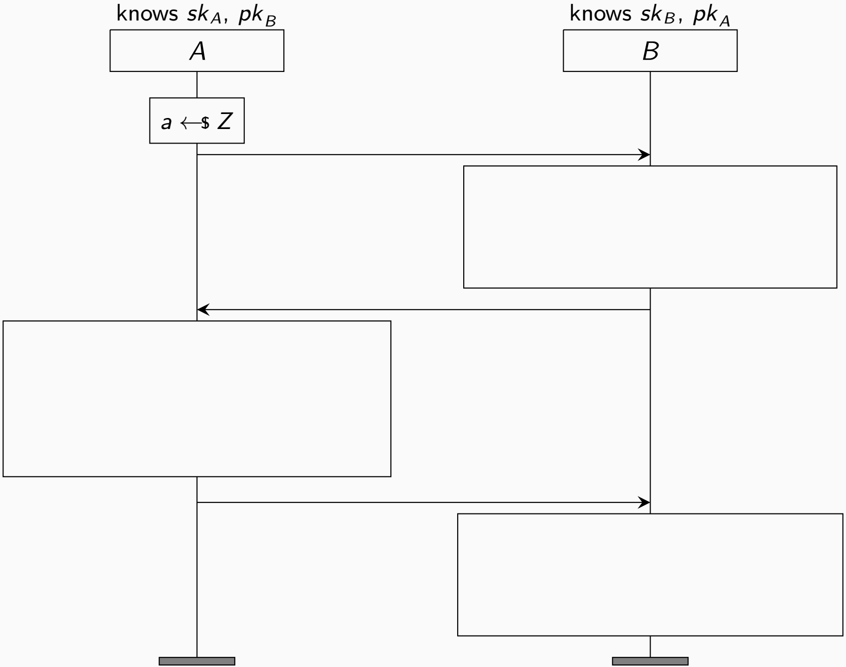
Diffie-Hellman Non-Interactive Key Exchange

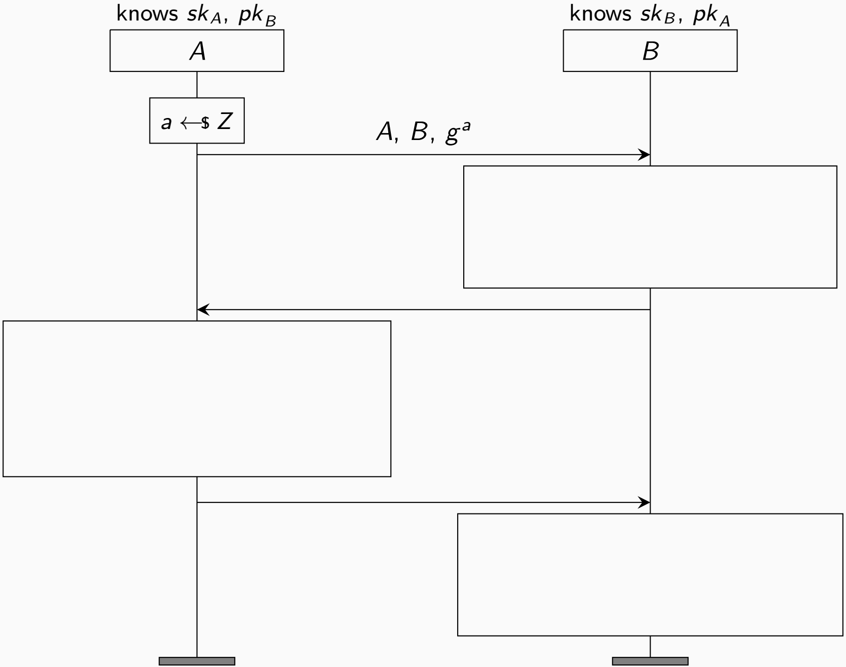
For simplicity, in a prime-order cyclic group $G = (\mathbb{Z}/p\mathbb{Z})^*$ of order p with generator g.
private keys: $a, b \xleftarrow{\$} Z = \{1, \ldots, p-1\}$
public keys: $g^a \mod p$, $g^b \mod p \in G$. $\quad (g^a, g^b$ in short)
DH shared secret: $(g^a)^b \mod p = (g^b)^a \mod p = g^{ab} \mod p$

Intuition: Knowing only the public keys, it is hard to recognize or compute the DH shared secret

Our Case Study: The Signed Diffie-Hellman Protocol


knows sk_A, pk_B
A
knows sk_B, pk_A
B


knows sk_A, pk_B
A
$a \xleftarrow{\$} Z$
knows sk_B, pk_A
B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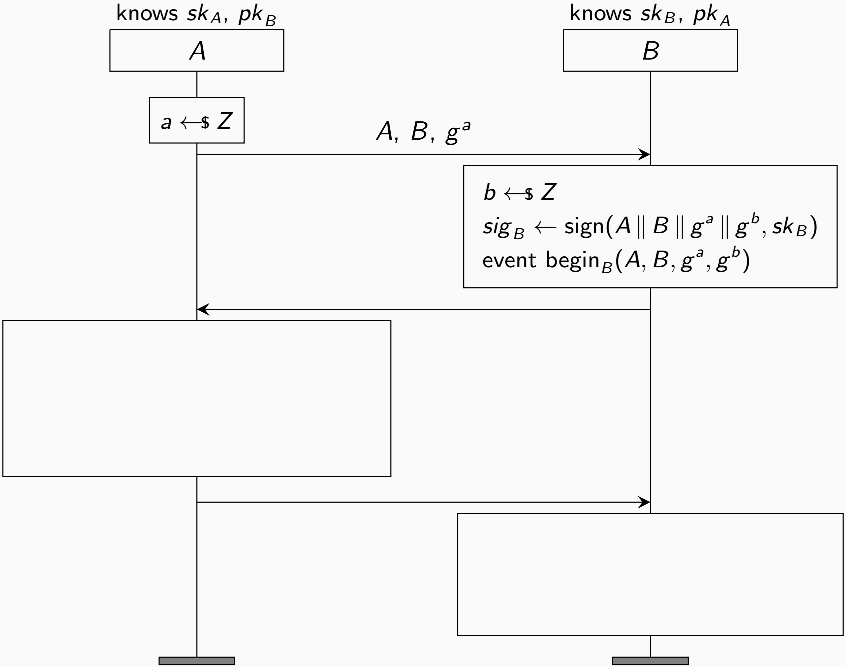
B
$a \leftarrow\!\!\$\ Z$
A, B, g^a



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$A,\ B,\ g^a$
$b \leftarrow\!\!\$\ Z$
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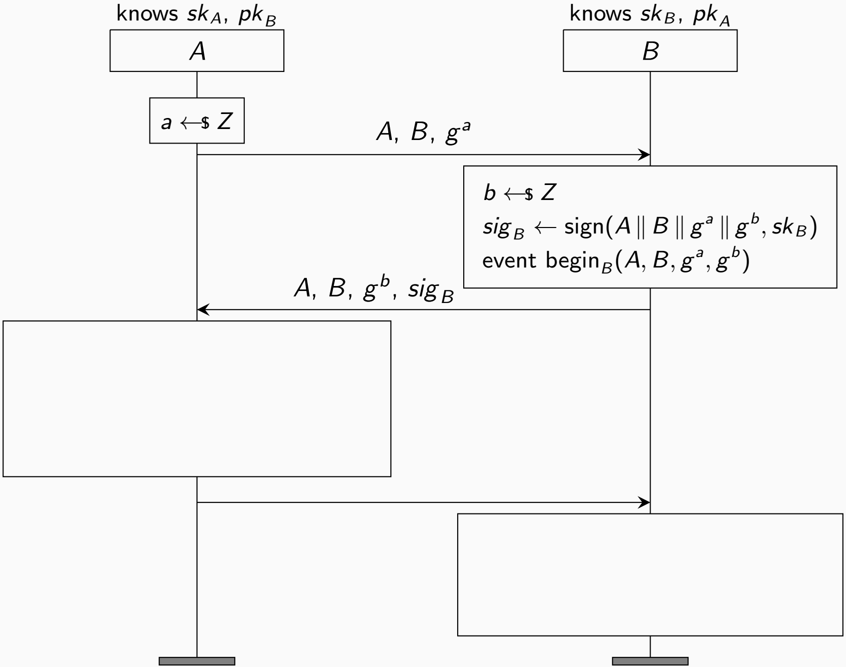
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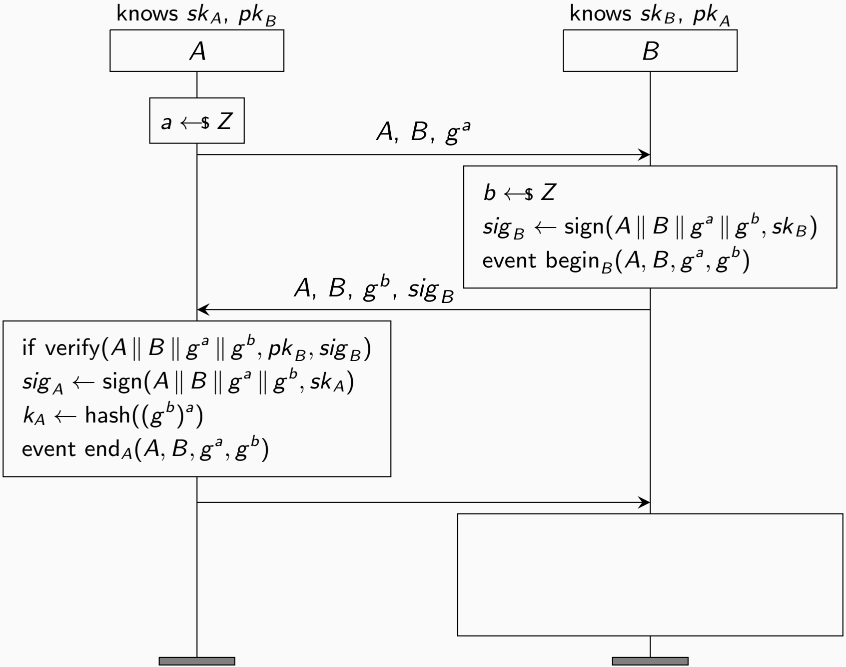
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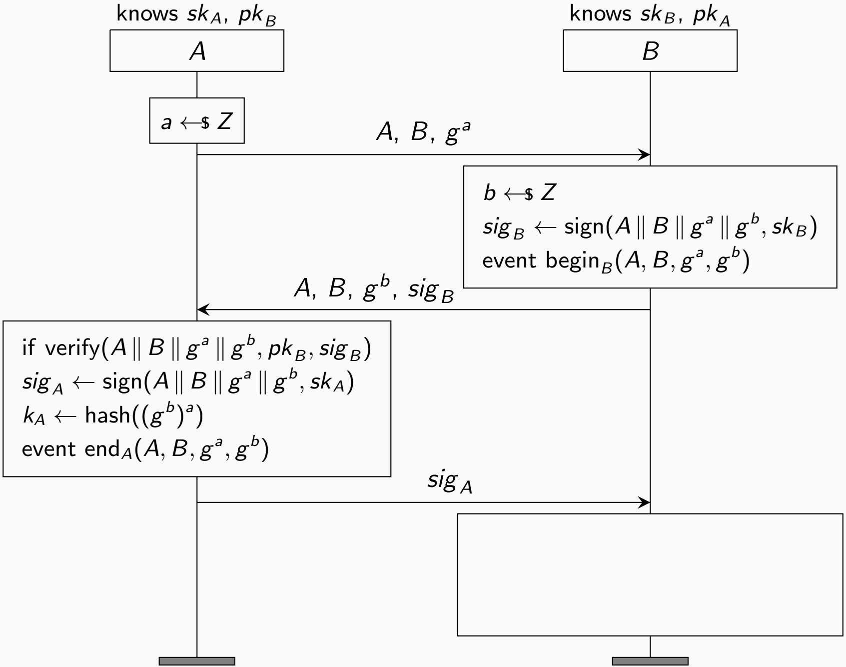
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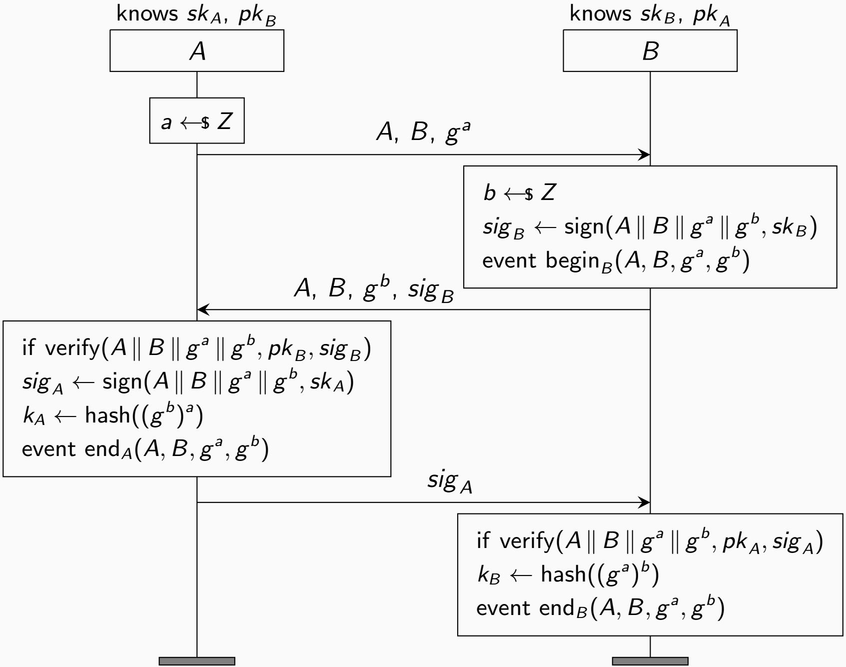
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Signed Diffie-Hellman: Security Properties

- The shared secrets k_A and k_B are secret (indistinguishable from random bitstrings of equal length)
- If A is convinced to have concluded a session with B using ephemerals g^a, g^b, then B actually started such a session
- If B is convinced to have concluded a session with A using ephemerals g^a, g^b, then A is likewise convinced

Cryptographic Assumptions

Cryptographic Assumptions

We use the following cryptographic assumptions to prove these security properties:

- hash is a random oracle
- $(\mathsf{sign}, \mathsf{verify})$ is a UF-CMA-secure probabilistic signature
- the CDH assumption holds in the group G

Random Oracle as Ideal Model for Hash Functions

A random oracle is an idealized random function that returns

- an independent uniformly random value on new input,
- the same value than before on previously seen input.

To model this, adversarial calls are observed by the security game through an oracle.

Definitional rewriting step done by CryptoVerif:

$\underline{\mathtt{ROM}_b}$

$\quad \mathcal{L} \leftarrow \emptyset$

$\quad \textbf{return } \mathcal{A}^{\mathtt{hash}_b}()$

$\underline{\mathtt{hash}_0(m)}$

$\quad \textbf{return } \mathsf{hash}(m)$

$\underline{\mathtt{hash}_1(m)}$

$\quad \textbf{if } \exists k : (m, k) \in \mathcal{L}$

$\qquad \textbf{return } k$

$\quad \textbf{else}$

$\qquad k \xleftarrow{\$} \{0,1\}^{\mathsf{hashlen}}$

$\qquad \mathcal{L} \leftarrow \mathcal{L} \cup \{(m, k)\}$

$\qquad \textbf{return } k$

Random Oracle – Preamble in CryptoVerif

Using a random oracle in CryptoVerif:

```
type hashfunction [fixed].

expand ROM_hash(
  hashfunction, (* type for hash function choice *)
  G,            (* type of input *)
  key,          (* type of output *)
  hash,         (* name of hash function *)
  hashoracle,   (* process defining the hash oracle *)
  qH            (* parameter: number of calls *)
).
```

Random Oracle Part – Macro Internals **[lib]**

The macro defines the hash function. The first parameter models the choice of the specific hash function: The adversary could call `hash`, but does not know the value the protocol uses for the 1st parameter.

```
fun hash(hashfunction, G): key.
```

The macro defines the oracle we must expose such that the adversary can use the RO:

```
param qH.

let hashoracle(hf: hashfunction) :=
  foreach ih <= qH do
  Ohash(x: G) :=
    return(hash(hf, x)).
```

It allows `qH` calls, a parameter that will appear in the final probability formula.

Random Oracle – Usage

In the setup of the initial game, we sample a random hash function

```
hf <-R hashfunction;
```

and use it in each call of hash:

```
kA <- hash(hf, gab);
```

We must include the process defined by the macro, such that the adversary can access the random oracle for its own calls:

```
run hashoracle(hf)
```

Random Oracle – Applying the Assumption

The hash function might be called within a replicated oracle:

```
foreach i <= N do (* ... *) kA <- hash(hf, gab) (* ... *)
```

Variables inside a replication are implicitly defined as arrays. Values are accessible via the replication index: `gab[i]`, `kA[i]`

Random Oracle – Applying the Assumption

The hash function might be called within a replicated oracle:

```
foreach i <= N do (* ... *) kA <- hash(hf, gab) (* ... *)
```

Variables inside a replication are implicitly defined as arrays. Values are accessible via the replication index: `gab[i]`, `kA[i]`

An array lookup using `find` can access specific values. Here is how to locally model the call by a random oracle (assuming that there is only this one call to `hash`):

```
foreach i <= N do (* ... *)

(find j <= N suchthat defined(gab[j], kA[j]) && gab = gab[j]
then kA[j]
else kA <-R key; kA)

(* ... *)
```

Random Oracle – Applying the Assumption


```
find j <= N suchthat defined(gab[j], kA[j]) && gab = gab[j]
then kA[j]
else kA <-R key; kA
```

When applying the RO assumption, CryptoVerif replaces each call of the hash function by an array lookup, comparing with *all* other inputs:

There will be one `find` branch per hash call.

In particular, the `hash` call in the `hashoracle` process will be replaced by a array lookup, comparing with all hash inputs used in the entire game.

Random Oracle – Applying the Assumption [lib]

```
foreach i <= N do
  (* ... *)
  kA <- hash(hf, gab)
  (* ... *)

let hashoracle(hf: hashfunction) :=
  foreach ih <= qH do
  Ohash(x: G) :=
    return(hash(hf, x)).
```

Random Oracle – Applying the Assumption


```
foreach i <= N do
  (* ... *)
  kA <- hash(hf, gab) (* before rewriting *)
  (* ... *)

let hashoracle(hf: hashfunction) :=
  foreach ih <= qH do
  Ohash(x: G) :=
    find j <= qH suchthat defined(x[j], k[j]) && x = x[j] then
      return(k[j])
    else find i <= N suchthat
                    defined(gab[i], kA[i]) && x = gab[i] then
      return(kA[i])
    else
      k <-R key;
      return(k).
```

UF-CMA-Secure Probabilistic Signature

- Unforgeability under Chosen Message Attack (UF-CMA)
- Security notion implemented by the appropriate CryptoVerif macro (simplified), where the adversary advantage

$$\mathrm{Adv}^{\mathsf{UF-CMA}}_{sign}(\mathcal{A}) = \left| \ \Pr\left[\mathtt{UF\text{-}CMA}_0(\mathcal{A}) \Rightarrow 1\right] - \Pr\left[\mathtt{UF\text{-}CMA}_1(\mathcal{A}) \Rightarrow 1\right] \right| \quad \text{is negligible.}$$

$\underline{\mathtt{UF\text{-}CMA}_b}$

$r \xleftarrow{\$} \mathcal{K}$

$\mathcal{L} \leftarrow \emptyset$

return $\mathcal{A}^{Sign, Verify_b}(pk(r))$

$\underline{\text{Oracle } \mathtt{Sign}(m)}$

$\mathcal{L} \leftarrow \mathcal{L} \cup \{m\}$

$\sigma \xleftarrow{\$} sign(m, sk(r))$

return σ

$\underline{\text{Oracle } \mathtt{Verify}_0(m, \sigma)}$

return $verify(m, pk(r), \sigma)$

$\underline{\text{Oracle } \mathtt{Verify}_1(m, \sigma)}$

return $m \in \mathcal{L} \land verify(m, pk(r), \sigma)$

Types and Probabilities for the Signature

Types define names for subsets of the bitstrings. The annotations restrict them on a high level.

```
type keyseed [large,fixed].
type pkey [bounded].
type skey [bounded].
type message [bounded].
type signature [bounded].
```

We define names for probabilities. They will appear in the final probability bound.

```
proba Psign.     (* breaking the UF-CMA property *)
proba Psigncoll. (* probability of collision between
                    independently generated keys *)
```

Using the Macro: UF-CMA-secure Signature

```
expand UF_CMA_proba_signature(
  (* types, to be defined outside the macro *)
  keyseed,
  pkey,
  skey,
  message,
  signature,
  (* names for functions defined by the macro *)
  skgen,
  pkgen,
  sign,
  verify,
  (* probabilities, to be defined outside the macro *)
  Psign,
  Psigncoll
).
```

Functions Defined by the Signature Macro

In this example, we use a *probabilistic* signature. The macro makes this transparent for us, by defining the seed type and a `sign` wrapper function.

```
fun skgen(keyseed):skey.
fun pkgen(keyseed):pkey.

fun verify(message, pkey, signature): bool.
fun sign_r(message, skey, sign_seed): signature.

letfun sign(m: message, sk: skey) =
  r <-R sign_seed; sign_r(m, sk, r).

equation forall m: message, r: keyseed, r2: sign_seed;
  verify(m, pkgen(r), sign_r(m, skgen(r), r2)) = true.
```

The Computational Diffie-Hellman (CDH) Assumption

- computing g^{xy} from g^x and g^y is hard
- a comparison $c = g^{xy}$ of an adversary-computed value c with g^{xy} is indistinguishable from **false** for the adversary
- using CDH in a game-rewriting step in CryptoVerif, in a simplified single-key version, where the adversary advantage

$$\mathrm{Adv}_G^{\mathrm{CDH}}(\mathcal{A}) = \left| \;\Pr\left[\mathtt{CDH}_0(\mathcal{A}) \Rightarrow 1\right] - \Pr\left[\mathtt{CDH}_1(\mathcal{A}) \Rightarrow 1\right] \right| \quad \text{is negligible.}$$

$\underline{\mathtt{CDH}_b}$
  $x, y \xleftarrow{\$} Z$
  return $\mathcal{A}^{\mathtt{DDH}_b}(g^x, g^y)$

$\underline{\mathtt{DDH}_0(c)}$
  return $c = g^{xy}$

$\underline{\mathtt{DDH}_1(c)}$
  return false

Diffie-Hellman Part I

```
type Z [large,bounded].
type G [large,bounded].

proba PCollKey1.
proba PCollKey2.
```

CryptoVerif's default library comes with several macros for groups.
We'll use a basic group in which some collision probabilities are negligible.

```
expand DH_proba_collision(
  G,          (* type of group elements *)
  Z,          (* type of exponents *)
  g,          (* group generator *)
  exp,        (* exponentiation function *)
  exp',       (* exp. func. after transformation *)
  mult,       (* func. for exponent multiplication *)
  PCollKey1,(* g^(fresh x) collides with indep. Y *)
  PCollKey2 (* g^(fr. x * fr. y) coll. w/ indep. Y *)
).
```

Diffie-Hellman Part II

[lib]

The macro defines the exponentiation function, a group generator, and equations for exponent multiplication. An extract:

```
fun exp(G, Z): G.
const g: G.

fun mult(Z, Z): Z.
equation builtin commut(mult).

equation forall a:G, x:Z, y:Z;
  exp(exp(a, x), y) = exp(a, mult(x, y)).
```

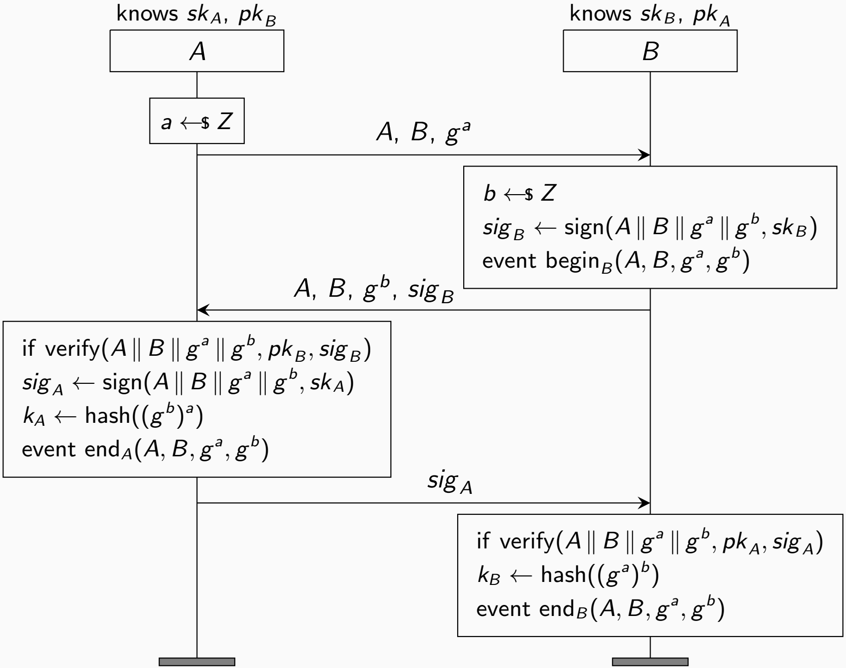
Diffie-Hellman Part III

Assumptions like CDH, DDH, GDH, ... must be instantiated with a separate macro. We use CDH, indicating the previously defined group:

```
proba pCDH. (* probability of breaking CDH in G *)
expand CDH(G, Z, g, exp, exp', mult, pCDH).
```

This macro implements a multi-key version of the version presented on the slides.

Semantics of the Security Queries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... if an adversary has a negligible probability of distinguishing keys k_A from uniformly random bitstrings of same length:

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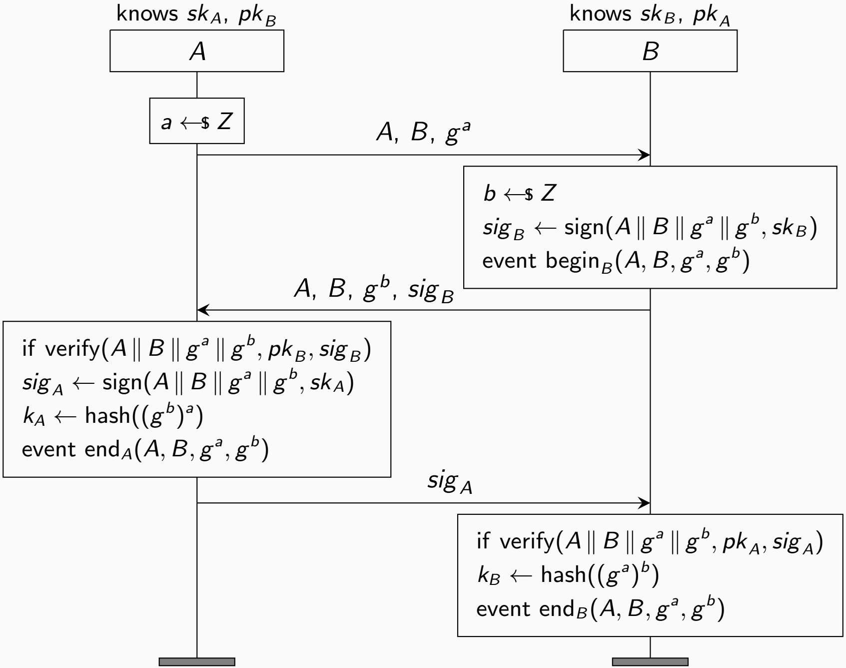
$$\begin{aligned}\mathrm{Adv}_{\mathrm{signedDH}}^{\textit{key-secrecy},k_A}(\mathcal{A}) = \big| \ & \Pr\left[\mathcal{G}_{real}(\mathcal{A}) \Rightarrow 1\right] \\ & - \Pr\left[\mathcal{G}_{random}(\mathcal{A}) \Rightarrow 1\right] \big|\end{aligned}$$

- where $\mathcal{G}_{real}$ is the original game (the initial game modeled in CryptoVerif), and
- in $\mathcal{G}_{random}$ (implicitly reasoned about by CryptoVerif), the keys k_A are replaced by independent uniformly random bitstrings of the same length.

This is different from usual pen-and-paper security notions where there is only one test session; here, all (honest) sessions are test sessions!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... if an adversary has a negligible probability of producing a sequence of events that violates the correspondence property:

$$\mathrm{Adv}^{auth,A}_{\mathrm{signedDH}}(\mathcal{A}) = \Pr\left[\begin{array}{l} \mathcal{A}^{Ostart,OA\cdot,OB\cdot,Opki,OH} : \mathcal{A} \text{ produces a sequence of events} \\ \text{such that not every } \mathrm{end}_B(A,B,g^a,g^b) \text{ is preceeded} \\ \text{by a distinct } \mathrm{end}_A(A,B,g^a,g^b) \end{array}\right]$$

Next Exercise Session

`(* It's your turn *)`

You should follow *instructions-practical-session-2.pdf* at:

`https://github.com/charlie-j/summer-school-2023/`

Feel free to refer to the cheatsheet, and to the slides of both sessions, and to ask questions!

Backup Slides

Interactive Mode

Include `interactive` in the proof environment to start the interactive mode:

```
proof {
  interactive
}
```

- `out_game "filename"` outputs the current game. Use a `.ocv` extension such that your editor highlights the syntax.
- `crypto assumption(function)` applies the assumption to the function. Example: `crypto rom(hash)`
- `success` tries to prove the queries
- `simplify` tries to simplify the current game
- `quit` leaves interactive mode and continues non-interactively.
- Ctrl+D ends the programme